**Formula Sheet 2**

**Theorem 3.7-> Theorem 5.7**

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# Chapter 3

## Definition 3.7:

* A random variable Y is said to have a binomial distribution based on n trials with success probability p if and only if.

## Theorem 3.7:

* Let Y be a binomial random variable based on n trials and success probability p. Then:

## Definition 3.8:

* A random variable Y is said to have a geometric probability distribution if and only if:

## Theorem 3.8:

* If Y is a random variable with a geometric distribution.

## Theorem 3.11

* Chebyshev’s Theorem Let Y be a random variable with mean µ and finite variance σ2. Then, for any constant k > 0,

# Chapter 4

## Definition 4.1

* Let Y denote any random variable. The distribution function of Y, denoted by F(y), is such that F(y) = P (Y ≤ y) for −∞ < y < ∞.

## Theorem 4.1

Properties of a Distribution function:

F(y)is a distribution function, then 1. F (−∞) ≡ Lim y→−∞F(y) = 0.

2. F (∞) ≡ Lim y→∞ F(y) = 1.

3. F(y) is a nondecreasing function of y. [If y1 and y2 are any values such that y1 < y2, then F(y1) ≤ F(y2).

## Definition 4.2

* A random variable Y with distribution function F(y) is said to be continuous if F(y) is continuous, for −∞ < y < ∞

## Definition 4.3

* Let F(y) be the distribution function for a continuous random variable Y. Then f (y), given by

Wherever the derivative exists, is called the probability density function for the random variable Y.

## Theorem 4.2

* Properties of a Density Function If f (y)is a density function for a continuous random variable, then.

1. f (y) ≥ 0 for all y, −∞ < y < ∞.

2. # ∞ −∞ f (y) dy = 1

## Definition 4.4

* Let Y denote any random variable. If 0 < p < 1, the pth quantile of Y, denoted by φp, is the smallest value such that P (Y ≤ φq) = F(φp) ≥ p. If Y is continuous, φp is the smallest value such that F(φp) = P (Y ≤ φp) = p. Some prefer to call φp the 100pth percentile of Y.

## Theorem 4.3

* If the random variable Y has density function f (y) and a < b, then the probability that Y falls in the interval [a, b] is.

## Definition 4.5

* The expected value of a continuous random variable Y is.

provided that the integral exists.

## Theorem 4.4

* Let g (Y) be a function of Y; then the expected value of g (Y ) is given by

provided that the integral exists.

## Theorem 4.5

* Let c be a constant and let g (Y), g1(Y), g2(Y), . . ., gk (Y) be functions of a continuous random variable Y. Then the following results hold:

1. E(c) = c.

2. E [cg (Y)] = cE [g (Y)].

3. E [g1(Y)+g2(Y)+· · ·+gk (Y)] = E [g1(Y)] +E [g2(Y)] +· · ·+E [gk (Y)]

## Definition 4.6

* If θ1 < θ2, a random variable Y is said to have a continuous uniform probability distribution on the interval (θ1, θ2) if and only if the density function of Y is.

## Definition 4.7

* The constants that determine the specific form of a density function are called parameters of the density function.

## Theorem 4.6

* If θ1 < θ2 and Y is a random variable uniformly distributed on the interval (θ1, θ2), then.

## Definition 4.8

* A random variable Y is said to have a normal probability distribution if and only if, for σ > 0 and −∞ <µ< ∞, the density function of Y is.

## Theorem 4.7

* If Y is a normally distributed random variable with parameters µ and σ, then:

# Chapter 5

## Definition 5.1

* Let Y1 and Y2 be discrete random variables. The joint (or bivariate) probability function for Y1 and Y2 is given by p (y1, y2) = P (Y1 = y1, Y2 = y2), −∞ < y1 < ∞, −∞ < y2 < ∞

## Theorem 5.1

* If Y1 and Y2 are discrete random variables with joint probability function p (y1, y2), then.

where the sum is over all values (y1, y2) that are assigned nonzero probabilities.

## Definition 5.2

* For any random variables , the joint (bivariate) distribution function F () is.

F ()) = P ( ≤ , ≤ ), − < < , − < < .

## Definition 5.3

* Let be continuous random variables with joint distribution function F (). If there exists a nonnegative function f (), such that.

for all −∞ < < ∞, −∞ < < ∞, then and are said to be jointly continuous random variables. The function f () is called the joint probability density function.

## Theorem 5.2

* If Y1 and Y2 are random variables with joint distribution function F (y1, y2), then.

1. F (−∞, −∞) = F (−∞, y2) = F (y1, −∞) = 0.

2. F (∞, ∞) = 1.

3. If y∗ 1 ≥ y1 and y∗ 2 ≥ y2, then F (y∗ 1, y∗ 2) − F(y∗ 1 , y2) − F(y1, y∗ 2 ) + F(y1, y2) ≥ 0.

## Theorem 5.2

* If Y1 and Y2 are jointly continuous random variables with a joint density function given by f (y1, y2), then 1. f (y1, y2) ≥ 0 for all y1, y2. 2. # ∞ −∞ # ∞ −∞ f (y1, y2) dy1 dy2 = 1

## Definition 5.4

* a Let Y1 and Y2 be jointly discrete random variables with probability function p(y1, y2). Then the marginal probability functions of Y1 and Y2, respectively, are given by
* b Let Y1 and Y2 be jointly continuous random variables with joint density function f (y1, y2). Then the marginal density functions of Y1 and Y2, respectively, are given by

## Definition 5.5

* If Y1 and Y2 are jointly discrete random variables with joint probability function p(y1, y2) and marginal probability functions p1(y1) and p2(y2), respectively, then the conditional discrete probability function of Y1 given Y2 is
* provided that p2(y2) > 0.

## Definition 5.6

* If Y1 and Y2 are jointly continuous random variables with joint density function f (y1, y2), then the conditional distribution function of Y1 given Y2 = y2 is

## Definition 5.7

* Let Y1 and Y2 be jointly continuous random variables with joint density f (y1, y2) and marginal densities f1(y1) and f2(y2), respectively. For any y2 such that f2(y2) > 0, the conditional density of Y1 given Y2 = y2 is given by
* and, for any y1 such that f1(y1) > 0, the conditional density of Y2 given Y1 = y1 is given by

## Definition 5.8

* Let Y1 have distribution function F1(y1), Y2 have distribution function F2(y2), and Y1 and Y2 have joint distribution function F(y1, y2). Then Y1 and Y2 are said to be independent if and only if
* for every pair of real numbers (y1, y2). If Y1 and Y2 are not independent, they are said to be dependent.

## Theorem 5.4

* If Y1 and Y2 are discrete random variables with joint probability function p (y1, y2) and marginal probability functions p1(y1) and p2(y2), respectively, then Y1 and Y2 are independent if and only if
* for all pairs of real numbers (y1, y2).
* If Y1 and Y2 are continuous random variables with joint density function f (y1, y2) and marginal density functions f1(y1) and f2(y2), respectively, then Y1 and Y2 are independent if and only if.
* for all pairs of real numbers (y1, y2).

## Theorem 5.5

* Let Y1 and Y2 have a joint density f (y1, y2) that is positive if and only if a ≤ y1 ≤ b and c ≤ y2 ≤ d, for constants a, b, c, and d; and f (y1, y2) = 0 otherwise. Then Y1 and Y2 are independent random variables if and only if
* where g(y1) is a nonnegative function of y1 alone and h(y2) is a nonnegative function of y2 alone.

## Theorem 5.6

* Let c be a constant. Then

## Theorem 5.7

* Let g(Y1, Y2) be a function of the random variables Y1 and Y2 and let c be a constant. Then